

# Comparative Analysis of Discrete Wavelet Transform and Complex Wavelet Transform For Image Fusion and De-Noiseing

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**ABSTRACT:** In various real life applications such as remote sensing and medical image diagnosis, image fusion plays imperative role and it is more popular for image processing applications. Because of inadequate nature of practical imaging systems the capture images or acquired images are corrupted from various noise hence fusion of image is an integrated approach where reduction of noise and retaining the original features of image is essential. Image fusion is the process of extracting meaningful visual information from two or more images and combining them to form one fused image. Discrete Wavelet Transform (DWT) has a wide range of application in fusion of noise images. Previously, real valued wavelet transforms have been used for image fusion. Although this technique has provided improvements over more inhabitant methods, this transform suffers from the shift variance and lack of directionality associated with its wavelet bases. These problems have been overcome by the use of a reversible and discrete complex wavelet transform (the Dual Tree Complex Wavelet Transform DT-CWT). This paper therefore introduces an alternative structure such as DT-CWT that is more flexible, highly directional and shift invariant which outperforms the conventional method in terms of PSNR and image quality improvement.

**Keywords -** Wavelet transform, Discrete Wavelet Transform (DWT), Dual-Tree Complex Wavelet Transform (DT-CWT), Image Fusion.

## I. INTRODUCTION

The successful fusion of images acquired from diverse instruments is of great importance in many applications, such as medical imaging, microscopic imaging, remote sensing, computer vision, and robotics. Image fusion can be defined as the processes by which several images, or some of their features, are combined together to form a single image. Image fusion can be performed at different levels of the information representation. Four different levels can be distinguished according to signal[1], pixel, feature and symbolic levels. To date, the results of image fusion in areas such as remote sensing and medical imaging are primarily intended for presentation to a human observer for easier and enhanced interpretation. Therefore, the perception of the fused image is of paramount importance when evaluating different fusion schemes. When fusion is done at pixel level the input images are combined without any pre-processing. Pixel level fusion algorithms vary from very simple, e.g. image averaging, to very complex, e.g. principal component analysis, pyramid based image fusion and wavelet transform fusion. Since the DWT has some limitation such as less directional selectivity, shift-invariance, aliasing, oscillation of wavelet coefficients and required higher computational cost because of highly redundant representation. These limitations of DWT are overcome by Dual Tree Complex Wavelet Transform(DT-CWT) up to the great extent. Here in this literature we have proposed a novel approach of DT-CWT based image fusion method. Even though it has complexity to implement but it gives best results than the Discrete Wavelet Transform based image fusion method. This paper is organized as follows: section 2 involves the Discrete wavelet transform and its limitations for image processing applications. The analysis and synthesis filter structures of Dual tree Complex wavelet transform and its highly directive and shift invariance properties has been discussed in section 3. The fusion process using DWT and DT-CWT is discussed in section 4. Section 5 gives the results of image fusion. Section 6 gives the conclusion.

## II. DISCRETE WAVELET TRANSFORM

### 2.1 Two Dimensional Discrete Wavelet transform

For two dimensional (2D) signals, e.g. images quad-tree structure shown in Fig.1. In a separable implementation, each level of the quad-tree comprises of two stages of filtering. The first stage filters and sub-samples the rows of the image, generating a pair of horizontal low pass and high pass sub-images. The second stage of the transform filters and sub-samples the columns of the filtered row signal to produce four sub-images, denoted B<sub>0</sub>, . . . , B<sub>3</sub>. This separable filtering implementation is the most efficient way to perform the 2D DWT [120]. Similar to the 1D case, for the multi-level transformation B<sub>0</sub> the low pass image obtained from the

previous level becomes the new input at the next level of the transform. Fig. 1 shows a two-level DWT decomposition of an image. The sub-band images  $B_1, B_2$  and  $B_3$  represent the detailed or higher pass wavelet coefficients representing the horizontal, diagonal and vertical components of the input signal. The transform can be further extended to high dimensions by applying filters to each dimension in turn. For  $m$  dimensional signals  $2m$  sub-band images are produced at each level. The DWT decomposition given in Fig. 1. These filters are the most favoured in image fusion and denoising application.

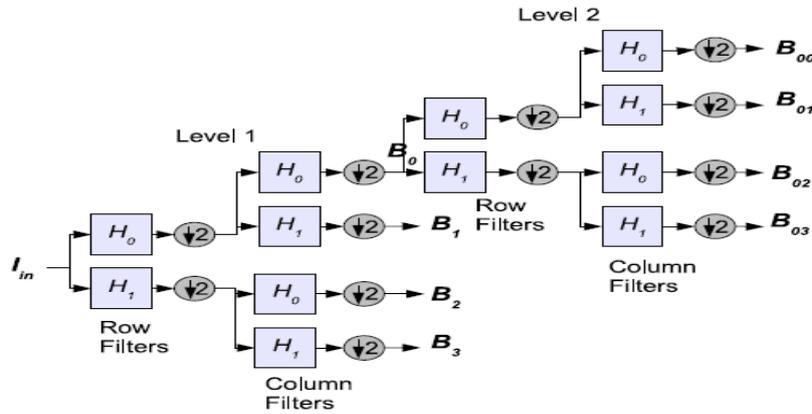


Fig: 1 Two-level DWT for a 2D signal.

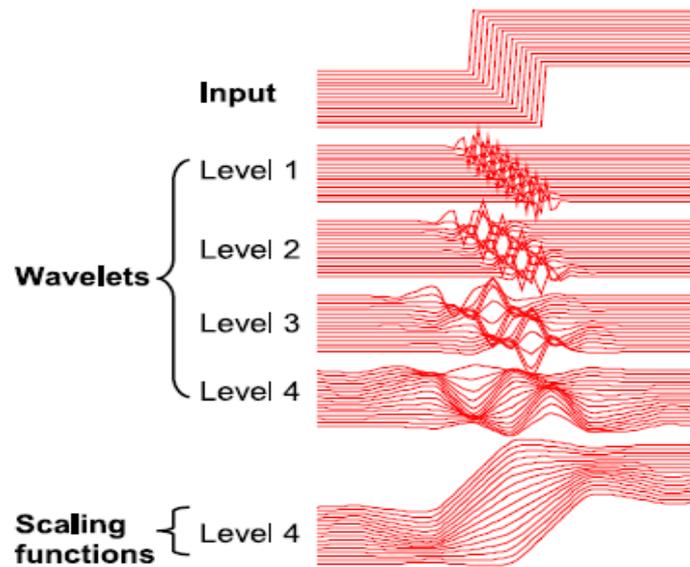


Fig.2 Two-level DWT decomposition of the input image using filters. In image, the upper right and bottom two quadrants are the level one sub-band images while their associated “child” sub-band images are nested inside them. The low pass image of scaling functions is located at the top left corner.

Although the DWT is widely used in image fusion and de-noising [4], its application to other image processing problems has been hampered by two principle disadvantages. These are:

### 2.1.1 Lack of shift invariance

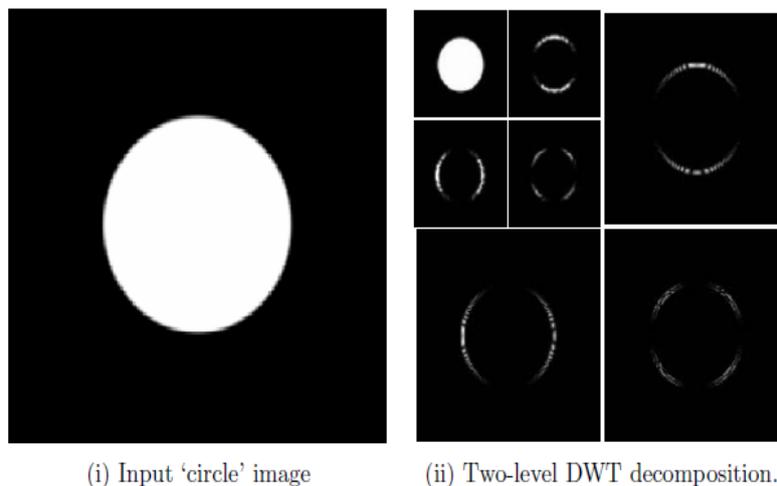
Strictly speaking this limitation should be considered as lack of shift covariance however, in the literature [3], the term shift invariance is used and this will be adopted here. A process is shift invariant if its output is independent of the absolute location of the data within the input to the process. With reference to the wavelet transform, the transform is shift invariant if the total energy in the sub-band image is unaffected by translations applied to the input. The shift dependency occurs as a result of the aliasing that is introduced by the down-sampling that follows each filtering operation. The un-decimated (remove the down sampling after the filtering) form of the DWT solves this problem but this is at the expense of large redundancy and increased computational expense. Fig.3 illustrates the shift dependence of the DWT. The input signal is a 1D step response shifted 16 times. Four levels of the DWT are taken and the sub-band signals associated with each is shown below the input signal. The level four scaling functions are shown on the bottom of the image. The shift dependence of the transform is evident from the varying energy in each of the sub-band signals. In image processing applications, the shift dependence of the DWT has limited its suit-ability for texture analysis. This is because in any given image, texture may present itself under any shift. If texture is to be characterized by its sub-band decomposition, then this decomposition needs to remain constant irrespective of the location of the texture within the image. As a result, any transform used to analyze texture should be as close to shift invariant as possible.



**Fig. 3** Shift dependence of the DWT. Input signal is 1D step function at sixteen different shifts. The sub-band and low pass signals associated with the four-level DWT decomposition are also shown. The shift dependence is evident from the fact that the energy in each sub-band at any given level varies with the shift in the input signal.

### 2.1.2 Poor directional selectivity

Separable filtering of the image rows and columns produces four sub-images at each level. These sub-band images are obtained using real filters which cannot distinguish between positive and negative frequency components. Therefore, each sub-band contains both positive and negative frequency components resulting in poor directional selectivity of the DWT. This inability to distinguish between positive and negative edge orientations increases the DWT unsuitability for texture analysis, given that textures are generally characterized by their frequency components. Figure 4 shows the poor directional selectivity of the DWT. The two level DWT of the ‘circle’ image (i) is shown in (ii). Level one sub-band images are placed on the boundary while the corresponding level two sub-band images are nested inside them. The final level scaling functions are shown at the top left hand corner of (ii). The intensity value in each sub-band images corresponds to the magnitude of the wavelet coefficient at that particular site. Each sub-band highlights either the horizontal, vertical or diagonal edge components of the input ‘circle’ image. The poor directional selectivity of the DWT is especially evident in the sub-band that contains the diagonal components of the circle (bottom right quadrant, bottom right of top left quadrant). These sub-bands contain both the diagonal edges of the circle making it impossible to distinguish between them.



**Fig. 4**, The poor direction selectivity of the DWT. The input “circle” image (i) and a two- level DWT decomposition (ii). Both positive and negative frequency components are represented in the diagonal sub-band (bottom right corner of (ii)) making it impossible to distinguish between them resulting in poor directional selectivity.

To address the lack of shift invariance and poor directional selectivity associated with the DWT, Kingsbury [119–121] developed the Dual Tree-Complex Wavelet Transform (DT-CWT). This will be discussed next.

### III. THE DUAL-TREE COMPLEX WAVELET TRANSFORM

The DT-CWT[2],[3] replaces the single tree structure of the DWT shown in Fig.1 with a dual tree of real-valued filters shown in Fig.5. These two parallel trees filter and down-sample the input signal in the same way as DWT but because there are two rather than one, the aliasing that resulted in shift dependency of the DWT is removed. At each level one tree produces the so called “real” part of the complex wavelet coefficients, while the other tree produces the “imaginary” part. The filters in each tree are real-valued and the notion of complex coefficients only appears when outputs from the two trees are combined. The addition of the second filter bank increases the redundancy of the transform to 2 : 1 for a 1D signal. For an mD signal, the redundancy of the transform is 2m : 1. As well as removing the shift dependence of the transform, the two tree structure also allows the positive and negative frequencies present in the original signal to be treated separately. At each level of the 2D DT-CWT, a low pass image and six sub-band images are produced. Each of the sub-band images contains the wavelet coefficients whose magnitude is proportional to any one of the  $\pm 15$ degree,  $\pm 45$ degree and  $\pm 75$ degree directional edges in the original image. Thus, the DT-CWT has associated with it good directional selectivity. This good directional selectivity

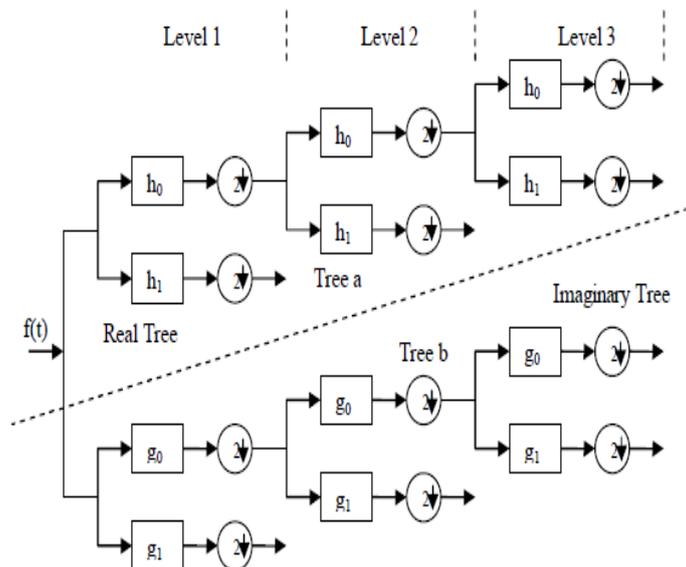


Fig. 5 Three-level DT-CWT Analysis Filter bank for decomposition of a 1D signal.

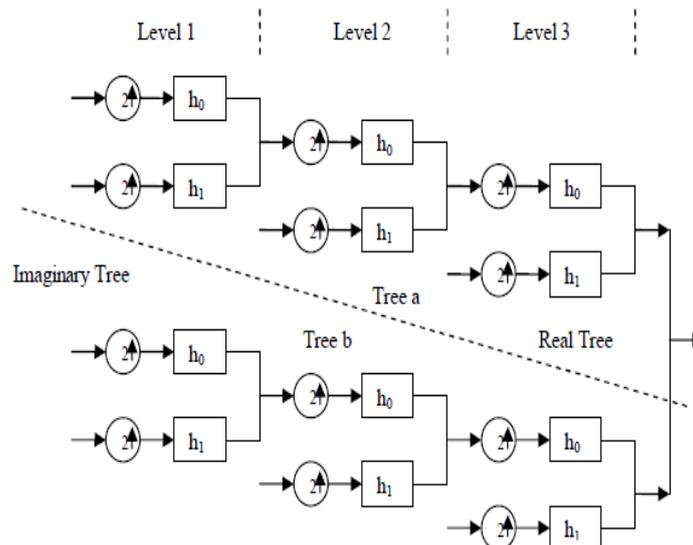
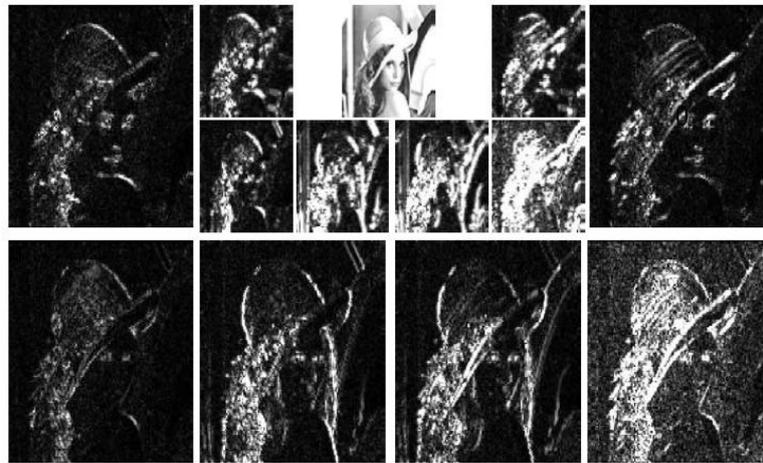
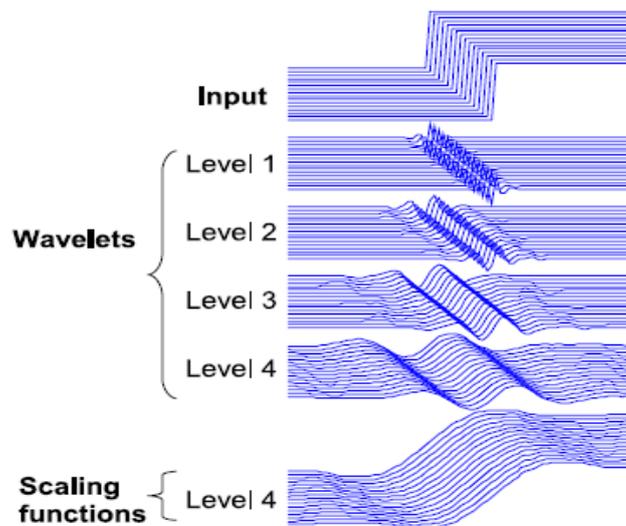


Fig. 6 Three-level DT-CWT Synthesis filter bank for reconstruction of 1D signal

Is advantageous for texture representation inherent with many image processing applications [2],[3]. Fig.6 shows a two-level DT-CWT decomposition of the sample image. The intensity value of each of the sub-band images is obtained from the absolute value of the complex wavelet coefficients in each sub-band image. Thus bright pixels in any of the sub-band images indicate a large frequency content for that particular orientation. Note the 4 : 1 redundancy for the 2D transform. This extra redundancy enables the properties of shift invariance and good directional selectivity to be associated with the DT-CWT. To demonstrate the shift invariance of the DT-CWT[3], an input signal consisting of a step function at sixteen different shifts is decomposed using a four level DT-CWT. Recall that if the transform is shift invariant, the energy in each sub-band should remain constant regardless of the shift in the input. Fig. 7 shows the sub-band signals associated with the DT-CWT. Since the energy with each sub-band signal at any given level remains constant regardless of shift, the DT-CWT is therefore shift invariant. The good directional properties of the DT-CWT are shown in Fig.9. The ‘circle’ image shown is the time decomposed using the DT-CWT. Unlike the DWT which combines positive and negative frequencies and produces three sub-band images at each level, the DT-CWT treats positive and negative frequencies separately and produces six sub-band images at each level. Each sub-band contains wavelet coefficients whose magnitude are proportional to one of the  $\pm 15$ degree,  $\pm 45$ degree,  $\pm 75$ degree directional orientations of the input signal. Because positive and negative orientations are treated separately, the DT-CWT provides greater directional selectivity than the DWT.



**Fig: 6, Two-level DT-CWT decomposition of the image given in figure . The level one sub-band images are located around the boundaries and the level two sub-band images are nested inside them. The level two low-pass image of scaling coefficients is located in the middle of row one.**



**Fig. 7, Shift invariance of the DT-CWT. Input signal consists of a step function at sixteen different shifts. The four-level DT-CWT decomposition is show below. At each level, the energy in each sub-band remains constant regardless of shift. This invariance of energy to shift implies that the transform is shift independent.**

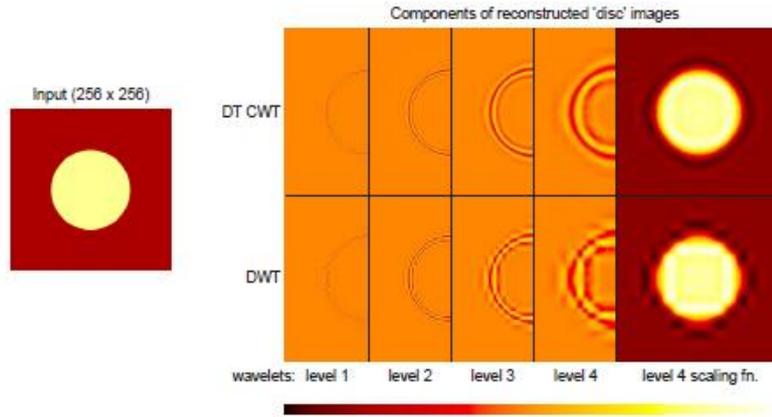


Fig. 8, 2-D Shift Invariance of DT CWT vs DWT ,Wavelet and scaling function components at levels 1 to 4 of an image of a light circular disc on a dark background, using the 2-D DT CWT (upper row) and 2-D DWT (lower row). Only half of each wavelet image is shown in order to save space.

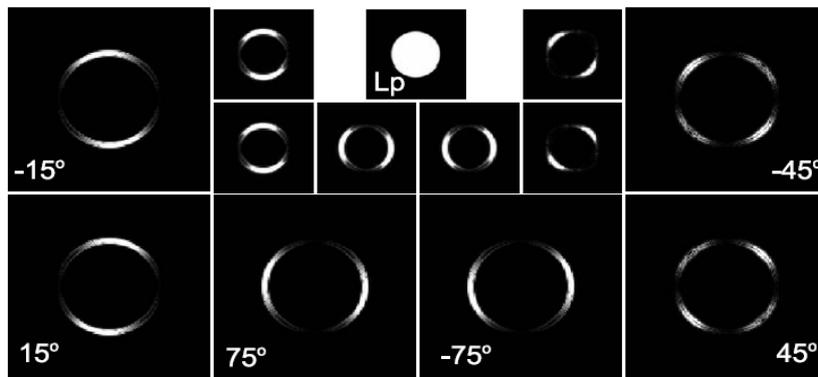


Fig. 9, The good directional properties of the DT-CWT are shown in the two-level DT-CWT decomposition of the ‘circle’ image shown in 3.9 (i). The level one sub-band images are located around the boundary and level two sub-band images are nested inside them. The level two low pass image of scaling functions is located at the top centre.

The six wavelets defined by oriented, as illustrated in fig.9. Because the sum/difference operation is orthonormal; this constitutes a perfect reconstruction wavelet transform. The imaginary part of 2D DT-CWT has similar basis function as the real part [3]. The 2-D DT-CWT structure has an extension of conjugate filtering in 2-D case. The filter bank structure of 2- D dual-tree is shown in fig.10. 2-D structure needs four trees for analysis as well as for synthesis. The pairs of conjugate filters are applied to two dimensions (x and y) directions, which can be expressed as:

$$(h_x + jg_x)(h_y + jg_y) = (h_x h_y - g_x g_y) + j(h_x g_y + g_x h_y) \quad (1)$$

The overall 2-D dual-tree structure is 4-times redundant (expensive) than the standard 2-D DWT. The *tree-a* and *tree-b* form the real pair, while the *tree-c* and *tree-d* form the imaginary pair of the analysis filter bank. Trees- (*a*~, *b*~) and trees- (*c*~, *d*~) are the real and imaginary pairs respectively in the synthesis filter bank similar to their corresponding analysis pairs [4].

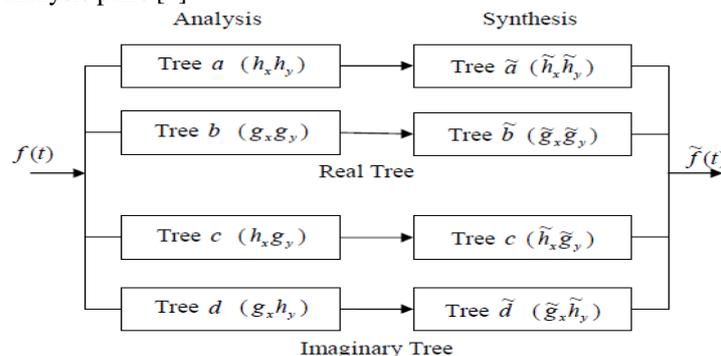


Fig. 10 Filter bank structure for 2-D DT-CWT

#### IV. FUSION PROCESS

The fusion process of two images using the DWT[4] is shown in figure (11). Here in this process the two images such as mask and bust are taken from two different sources which is decomposed first and then fused to convert it in synthesized image with help of DWT. In fig.12 the two images were taken from a multi-focus set, i.e. two registered images of same scene each with a different camera focus. This figure shows that the coefficients of each transform have significantly different magnitudes within the regions of different focus. A simple “maximum selection” was used to generate the combined coefficient map. This effectively retains the coefficients of “in focus” regions within the image. This inverse wavelet transform is then applied to the combined coefficient map to produce the fused image which in this case shown an image retaining the focus from the two input images.

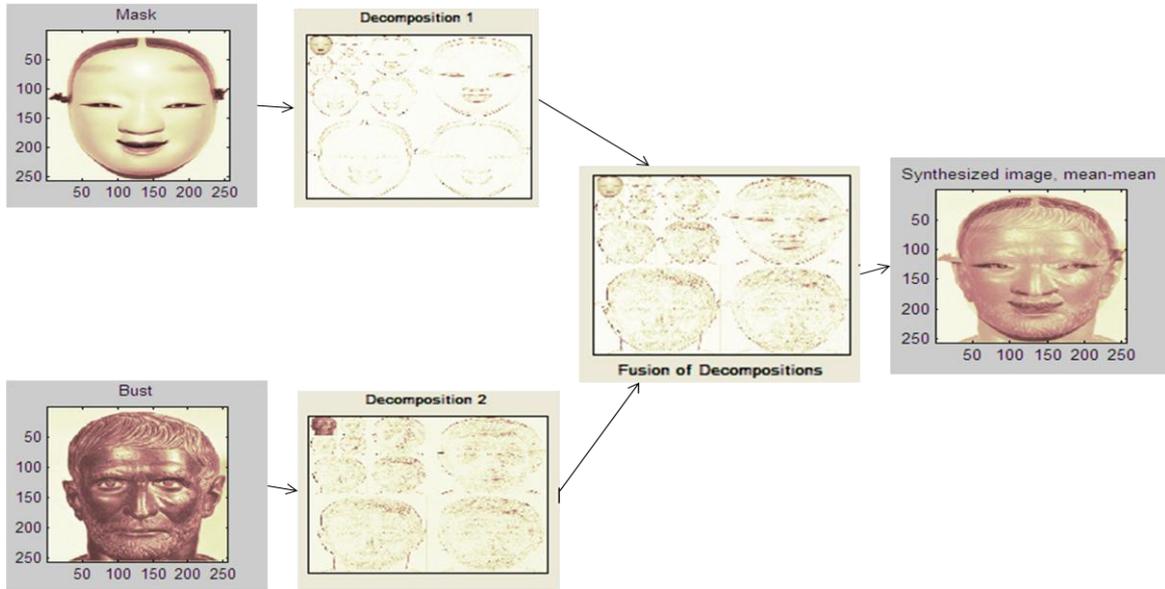


Fig. 11: Image fusion process using the DWT and two registered Mask and Bust images.

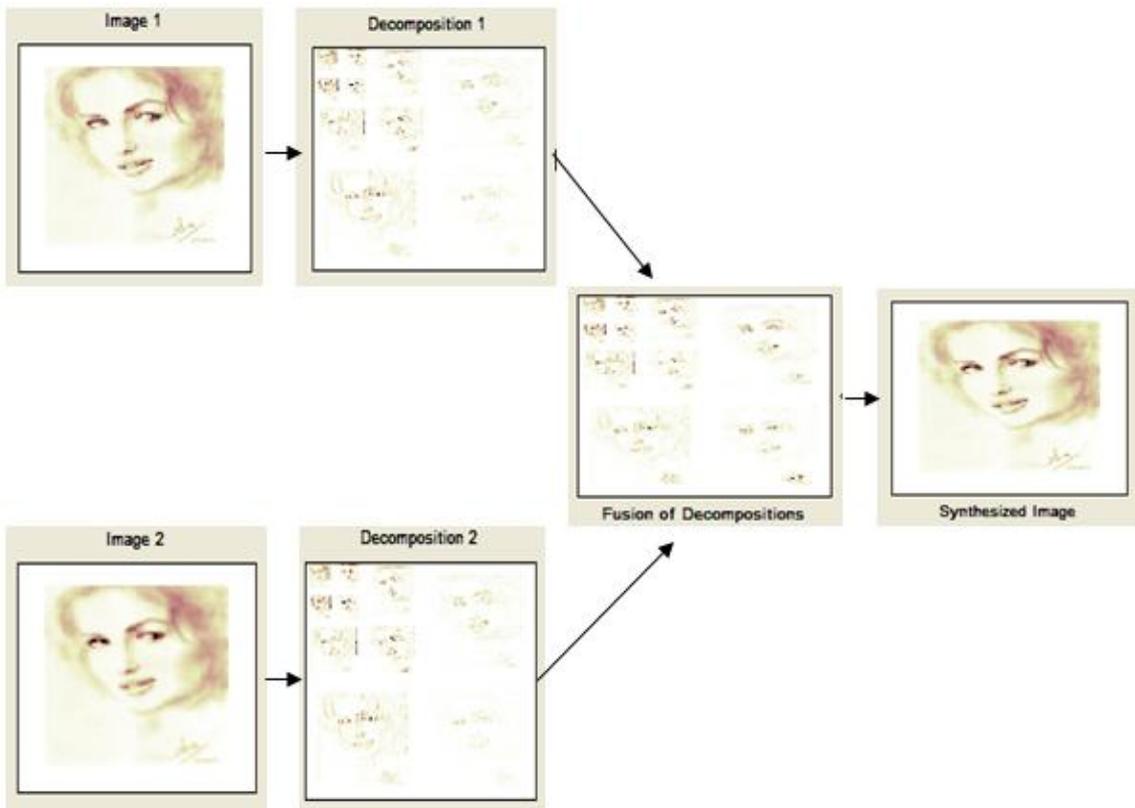


Fig. 12: Image fusion process using the DWT and two registered multi-focus Katrina Kaif images.

The dual-tree complex wavelet transform (DT-CWT) [2,10] iteratively applies separable spatial filters to produce frequency sub-bands in a similar way to the classical discrete wavelet transform. The prime motivation for producing the dual-tree complex wavelet transform was shift invariance. In a normal wavelet decomposition small shifts of the input signal are able to move energy between output sub bands. This is a result of the sub sampling necessary for critical decimation. The shift invariant discrete wavelet transform was an initial attempt to integrate shift invariance into a DWT by discarding all sub sampling. Shift variance can also be achieved in a DWT by doubling the sampling rate. This is effected in the DT-DWT by eliminating the down sampling by 2 after the first level of filtering. Two fully decimated trees are then produced by down sampling, effected by taking first the even and then the odd samples after the first level filters. To get uniform intervals between the two trees samples, the subsequent filters in one tree must have delays that are half a sample different. For linear phase, this is enforced if the filters in one tree are even and the filter in the other are odd. Additionally, better symmetry is achieved if each tree uses odd and even filters alternately from level to level. The filters are chosen from perfect reconstruction biorthogonal set and the impulse response can be considered as the real and imaginary parts of a complex wavelet[2]. Application to images is achieved by separable complex filtering in two dimensions. Fig.13 demonstrates the fusion of two images using the complex wavelet transform[11],[12]. The areas of the images more in focus give rise to larger magnitude coefficients within that region. A simple maximum selection scheme is used to produce the combined coefficient map. The resulting fused image is then produced by transforming the combined coefficient map using the inverse complex wavelet transform. The wavelet coefficient images show the orientation selectivity of the complex wavelet sub bands. Each of the clock hands which are pointing in different directions are picked out by differently oriented sub bands. We have implemented the same three fusion rules with the complex wavelet transform as discussed in section 4.1. A complex wavelet transform was used with the filters given in [Kingusbury,2000] designed for improved shift invariance.

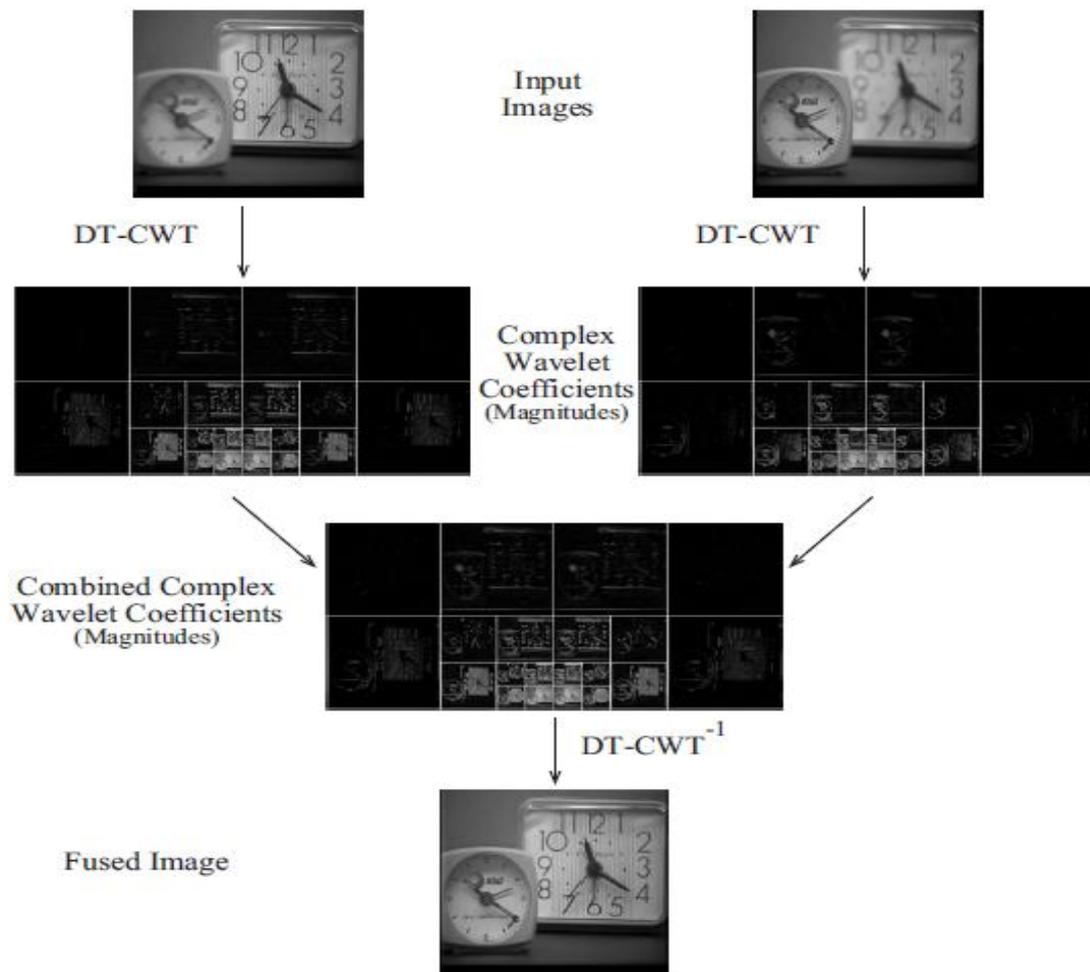


Fig. 13. The image fusion process using the DT-CWT and two registered multi-focus clock images.

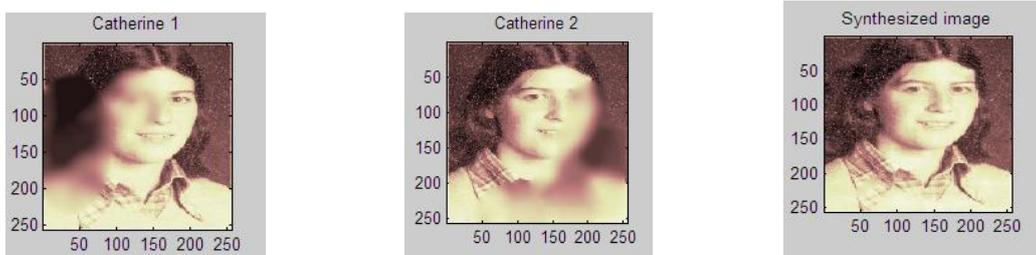
a. Implementation of Fusion Rule

Three previously developed fusion rule[7],[8] schemes were implemented using discrete wavelet transform based image fusion:

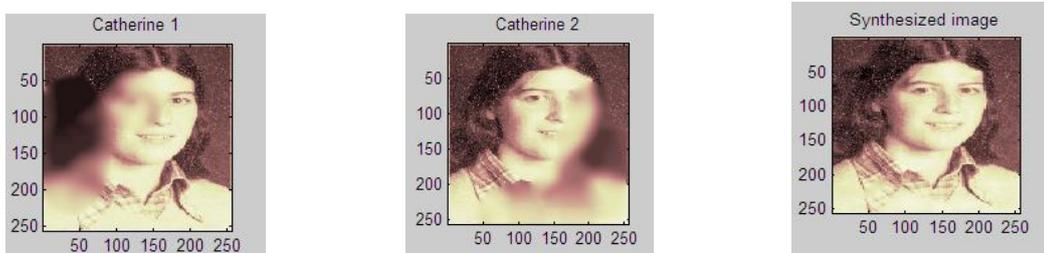
- **Maximum Selection (MS) Scheme:** This MS scheme picks the coefficient in each sub-band with the largest magnitude.
- **Weighted Average (WA) Scheme:** In this scheme a normalized correlation between the two images sub-bands over a small local area. The resultant coefficient for reconstruction is calculated from this measure via a weighted average of the two images coefficients.
- **Window based Verification (WBV) Scheme:** It creates a binary decision map to choose between each pair of coefficients using a majority filter[1].

**V. RESULT**

The performance of DWT fusion method and DT-CWT fusion methods are compared by considering two different color images of Catherine 1(Left corrupted) and Catherine 2 (Right Corrupted) . The Synthesized image after DWT fusion method and DT-CWT based fusion methods are a shown in figures (13) and (14) respectively. It has been observed from figure(14) that the DT-CWT fusion techniques provide better quantitative and qualitative results than the DWT at the expense of increased computation. The DT-CWT method is able to retain edge information without significant ringing artifacts. It is also good at faithfully retaining textures from the input images. All of these features can be attributed to the increased shift invariance and orientation selectivity of the DT-CWT when compared to the DWT. Hence it has been demonstrated with the help of results that DT-CWT is an essential tool for image fusion and de-noising. Due to improved directive and shift invariant properties of DT-CWT fusion method outperforms the DWT fusion method.



**Fig. 13: Image Fusion and De-noised Image Using DWT**



**Fig. 14: Image Fusion and De-noised Image Using DT-CWT**

Comparison and performance evaluation of different test images using DWT and DT-CWT is tabulated below. Table 1 show the Peak Signal-to-Noise-Ratio (PSNR) and Normalized Cross Correlation (NCC) of different images using DWT and DT-CWT.

**Table1: Comparison evaluation between DWT and DT-CWT for three different images.**

Images	Katrina Kaif Image		Catherine Image		Mask Image	
	PSNR	NCC	PSNR	NCC	PSNR	NCC
<b>DWT</b>	<b>21.69</b>	<b>0.87</b>	<b>19.64</b>	<b>0.86</b>	<b>21.39</b>	<b>0.88</b>
<b>DT-CWT</b>	<b>33.54</b>	<b>0.97</b>	<b>31.46</b>	<b>0.98</b>	<b>32.18</b>	<b>0.96</b>

### Comparative Analysis of DWT and DT-CWT

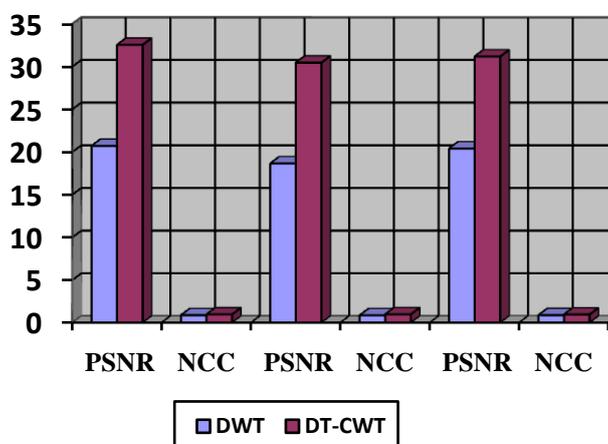


Fig 15: Comparative analysis of DWT and DT-CWT of three different images

### VI. CONCLUSION

The objective of this work has been to propose the comparative analysis between newly developed wavelet transform fusion methods with the existing fusion techniques. For an effective fusion of images a technique should aim to retain important features from all input images. These features often appear at different positions and scales. Multi resolution analysis tools such as the complex wavelet transform are ideally suited for image fusion. Simple DWT method for image fusion have produced limited results and suffers from poor directional ability and shift variant property. The DT-CWT fusion technique of noisy Catherine 1 and Catherine 2 provides better results than the DWT fusion technique as depicted in fig.13 and fig. 14. The DT-CWT based fusion method is able to retain important edge information without significant humming artifacts. DT-CWT provides increased shift-invariance and orientation selectivity when compared to the DWT. This is demonstrated and furnished in table1 shown above. The above fig.15 shows the analysis and performance of DWT and DT-CWT for three test images of Katrina Kaif, Mask image and Catherine image.

### ACKNOWLEDGMENT

The authors would like to thank N. G. Kingsbury, from the University of Cambridge, for providing the DT-CWT code for Matlab and Oliver Rockinger from Metapix, for providing the image fusion toolbox for Matlab without that it is not possible for us to proceed our research in this field.

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